MSE-238
Structure of Materials

Week 2 – crystallography I

Spring 2025

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Overview

- Lattice and motif
- Symmetry (as a first introduction, more next week)
- The hard sphere model: coordination and interstitial sites
- Crystals formed by metals
- Ionic and covalent crystals

- → Hammond Chapter 1

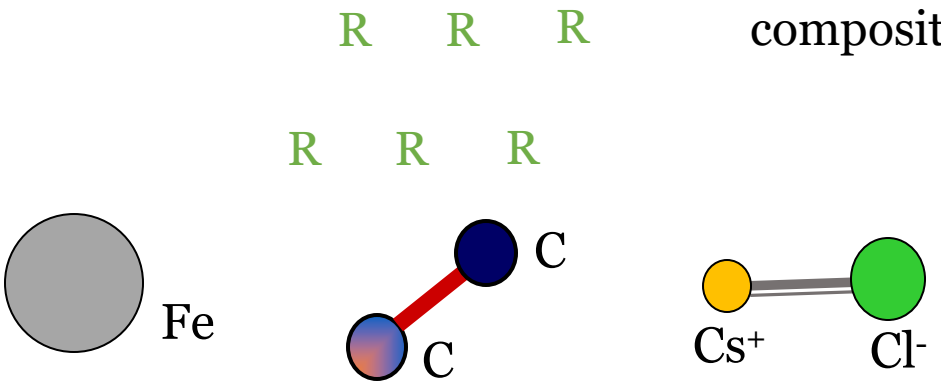
Crystalline material

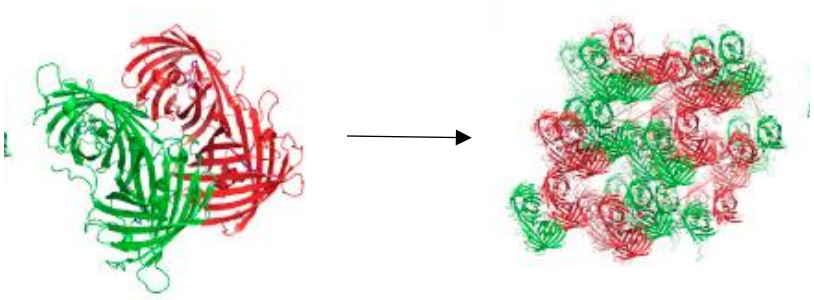
- recap Definition: A **crystalline material** is a material characterized by a **regular arrangement of atoms** or group of atoms over "large" distances, i.e. a crystal displays translational symmetry over a long range.
  In a crystal it is always possible to **identify a group of atoms or molecules** (can be also one atom) that **repeats** itself periodically **on a grid** in space.

Crystal = Motif + Lattice

- in 2D

Motif: repeating "unit of pattern"
here represented by R
atom, group of atoms, molecules → chemical composition





photoactivatable fluorescent protein
h41pkve6 → can be crystalized

Crystalline material

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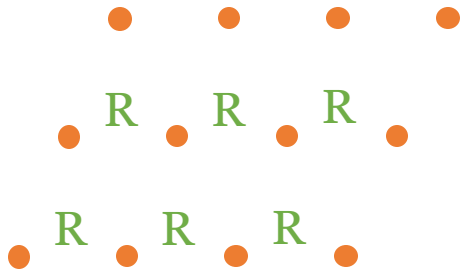

Motif: repeating "unit of pattern"
here represented by R
atom, group of atoms, molecules → chemical composition

Lattice: (imaginary) grid, intersections
→ lattice points
can be placed anywhere, but always in the same position with respect to the motif
in each point of the lattice, the crystal appears identical, **translational symmetry** in between

Crystalline material

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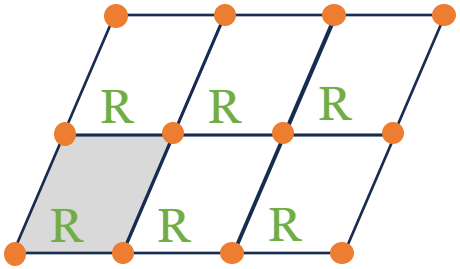
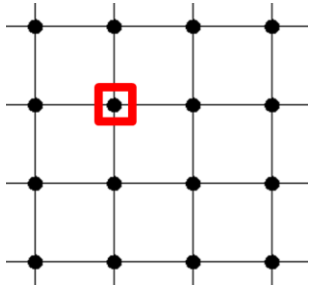


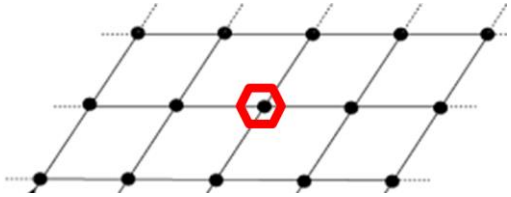
unit cell: arbitrary ways of joining up the lattice points.
primitive unit cell: smallest possible unit, contains one lattice point (here 4 times $1/4^{th}$)
conventional unit cell: larger unit cell, chose to reflect the crystal's symmetry better
contain more than one lattice points
makes visualization and classification of crystal structure easier

Crystal symmetry

- symmetry present in the crystal is determined by
  - how the translation is done (the lattice)

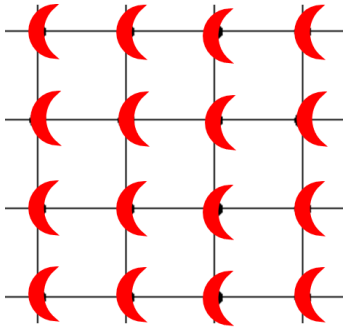


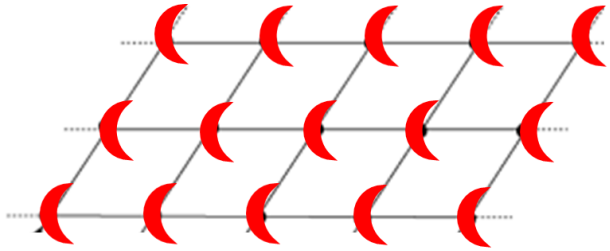
4-fold
rotation axis



6-fold
rotation axis

  - and the character of the motif

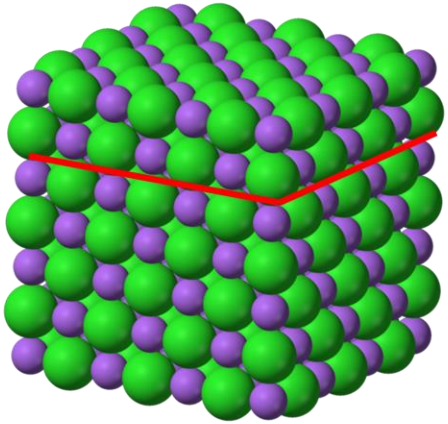




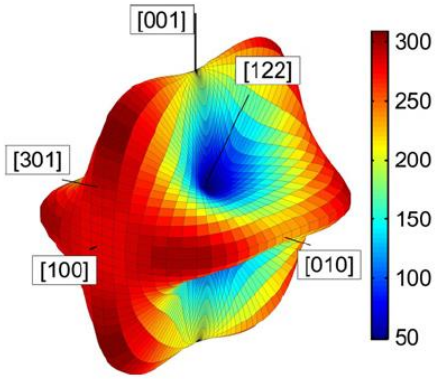
only 1-fold symmetry left!

Crystal symmetry

- Symmetry plays an important role in material science. Most material properties are anisotropic which means that their properties depend upon direction or angular orientation of crystals. Examples are chemical resistance, the ability to get charged when stressed (piezoelectricity) or fracture toughness.



in crystal of NaCl cleavage along planes parallel to cube faces

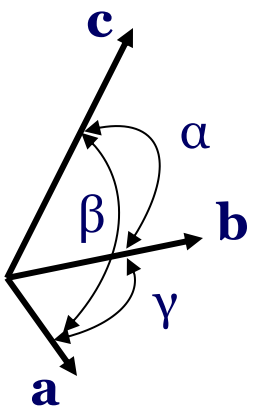


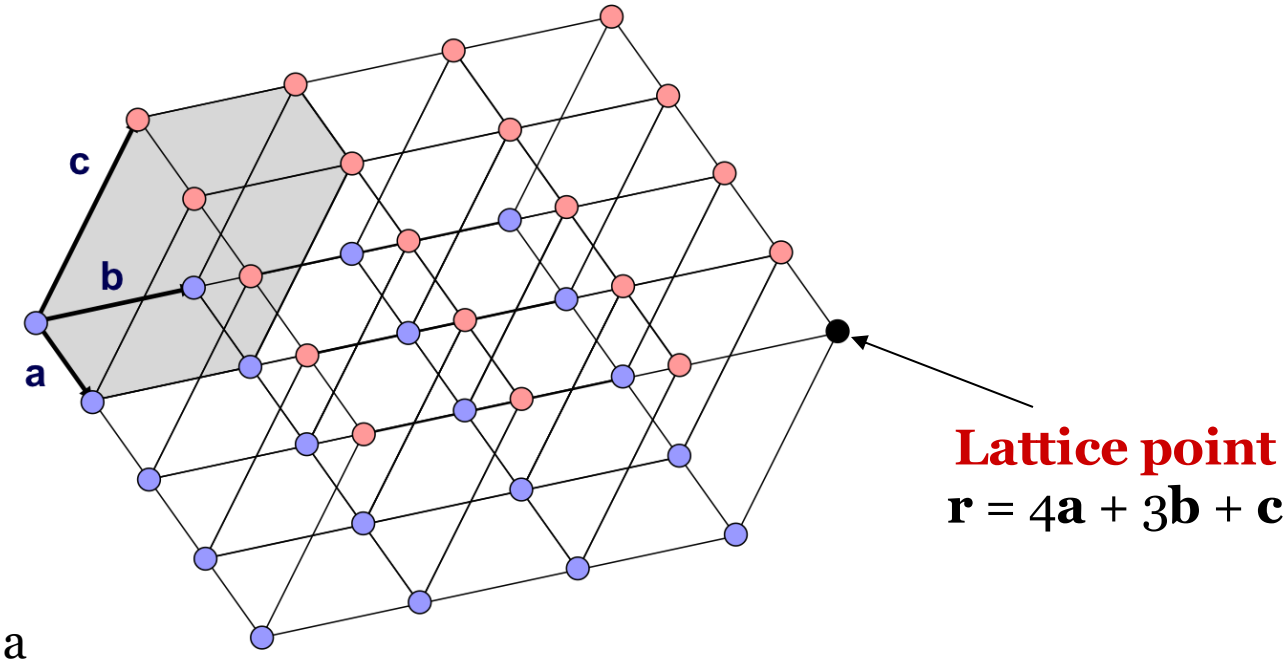
modulus of elasticity for single crystal elastic constant of cementite (Fe^3C)

Weisser et al Acta Materialia 59 (2011) 4448

Crystalline material

- in 3D: Bravais Lattice



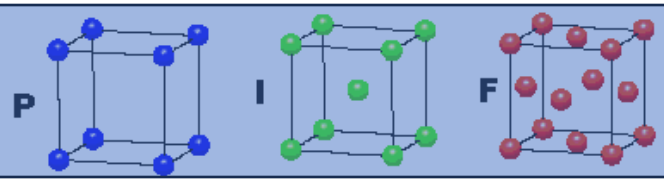


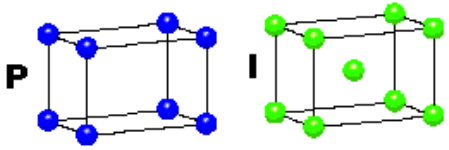
set of 3 vectors form the basis: every lattice point is a linear combination with relative integers as coefficients

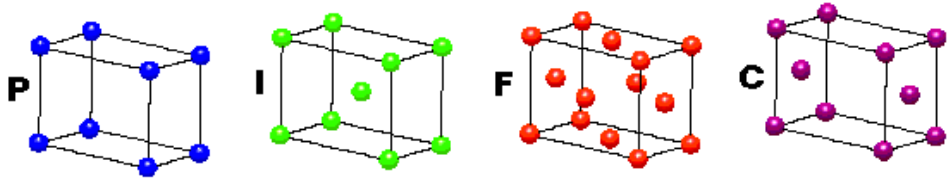
→ see more in 2 weeks: Miller indices

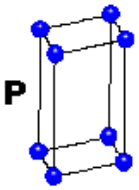
7 crystal systems – 14 Bravais Lattice

Cubic	a = b = c a = b = g = 90°
Tetragonal	a = b ≠ c a = b = g = 90°
Orthorhombic	a ≠ b ≠ c a = b = g = 90°
Hexagonal	a = b ≠ c a = b = 90°; g = 120°
Monoclinic	a ≠ b ≠ c a = g = 90° ≠ b
Triclinic	a ≠ b ≠ c a ≠ b ≠ g

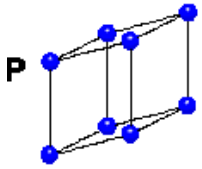




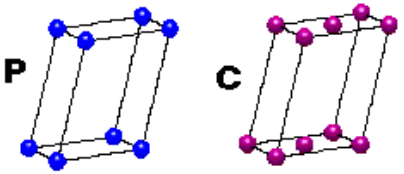


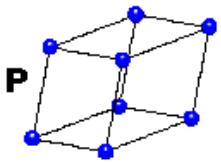


Trigonal or rhombohedric



a = b = c
a = b = g ≠ 90°





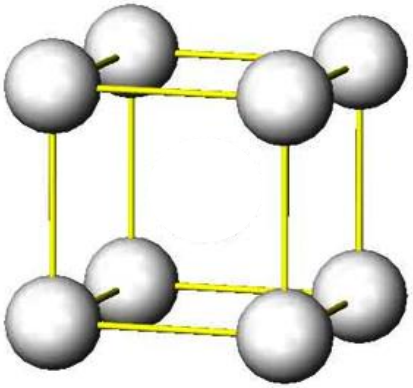
7 classes / 14 Bravais lattice

P : primitive
I : centered
F : face centered
C : base centered

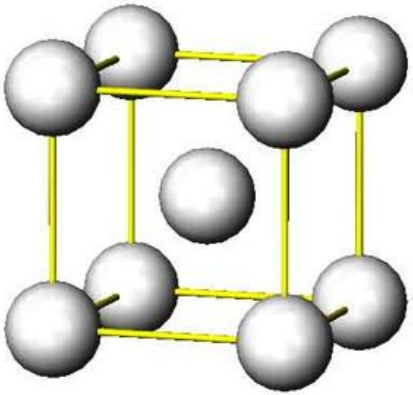
http://www.chem.ox.ac.uk/icl/heyes/structure_of_solids/lecture1/Bravais.gif

The cubic system

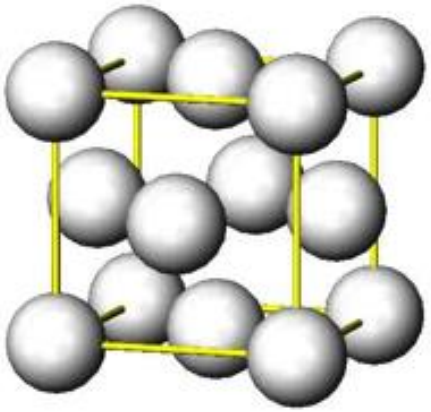
- The cubic system has three Bravais lattices



Primitive Cubic



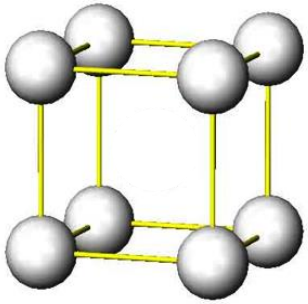
Body-centered Cubic



Face-centered Cubic

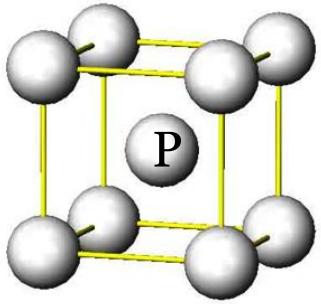
The cubic system: unit cells

primitive unit cell: smallest possible unit, contains one lattice point
conventional unit cell: larger unit cell, chose to reflect the crystal's symmetry
contain more than one lattice points (and motifs)
makes visualization and classification of crystal structure easier



Primitive cubic
basis: (**a**,**b**,**c**)

(**a**,**b**,**c**) is not a basis for BCC, P has the coordinates (1/2, 1/2, 1/2)

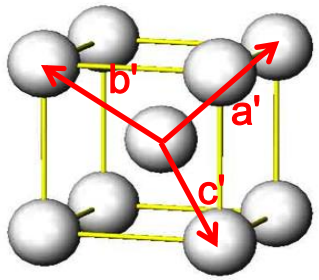


$$\vec{a}' = \frac{1}{2}(-\vec{a} + \vec{b} + \vec{c})$$

$$\vec{b}' = \frac{1}{2}(\vec{a} - \vec{b} + \vec{c})$$

$$\vec{c}' = \frac{1}{2}(\vec{a} + \vec{b} - \vec{c})$$

Body-centered cubic
basis: (**a'**,**b'**,**c'**)



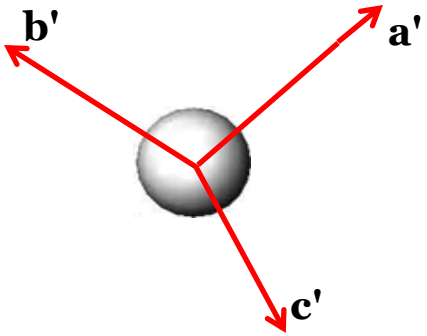
The cubic system: body-centered cubic (bcc)

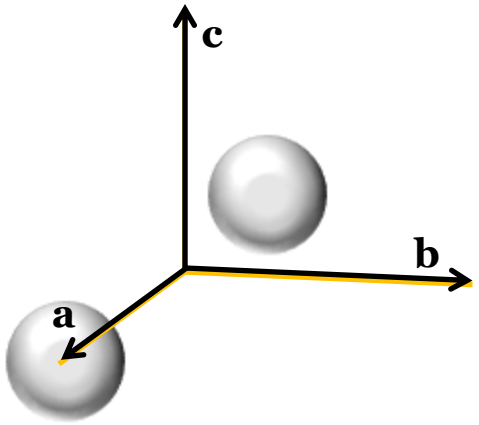
Primitive cell

- Find the motif;
- Translate it along the BCC lattice;

Convetional cell

- Find the motif;
- Place it within the Conventional cell;
- Translate the cell along the orthonormal basis **a**, **b** and **c** of the cubic system;





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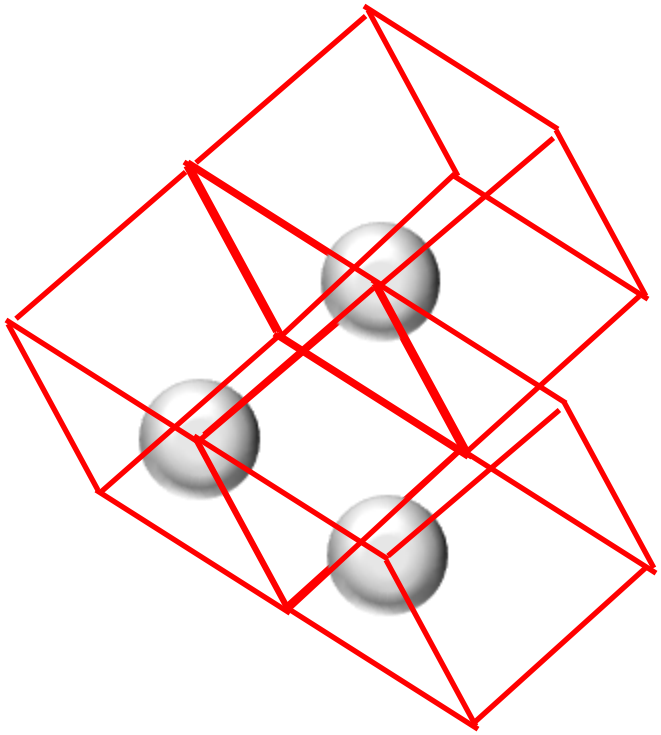

Convetional cell

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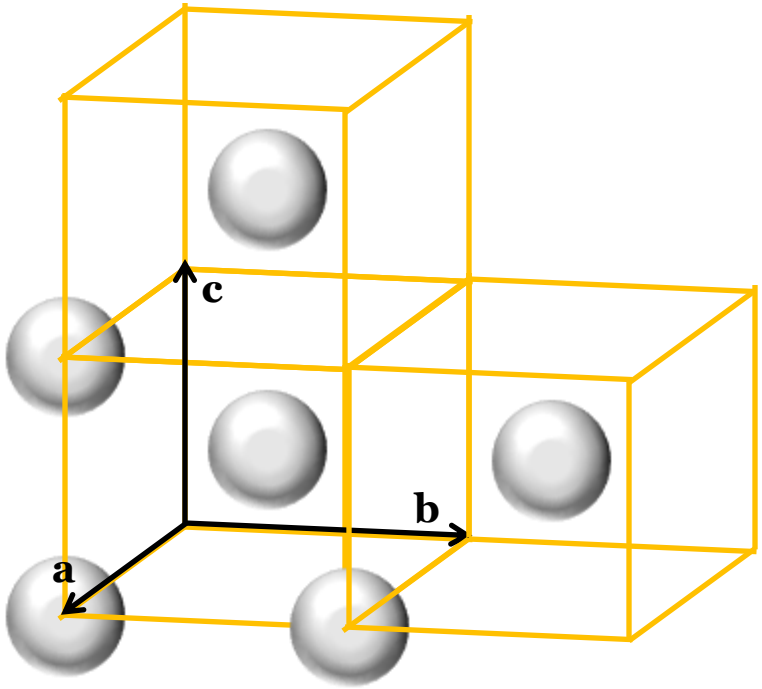

The cubic system: body-centered cubic (bcc)

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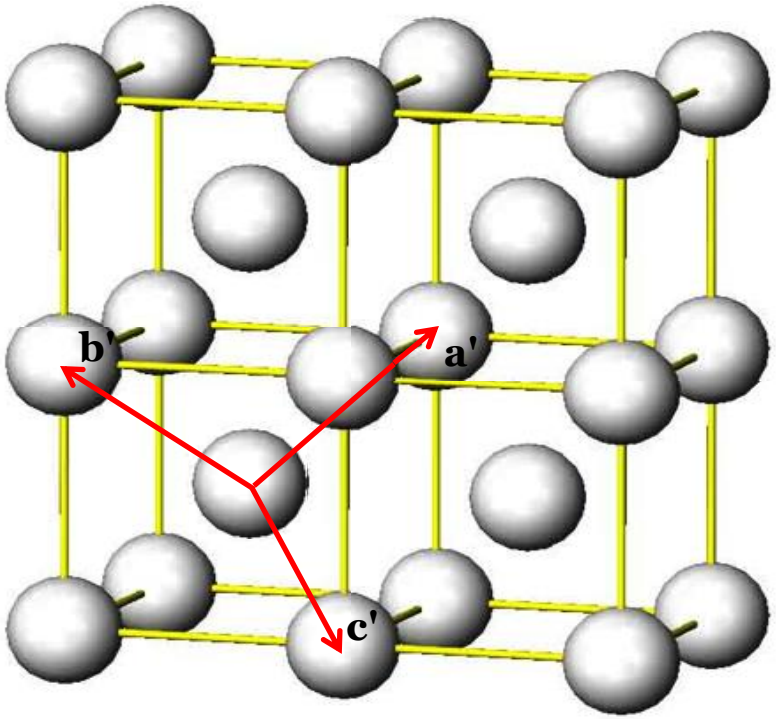

Convetional cell

- Find the motif;
- Place it within the Conventional cell;
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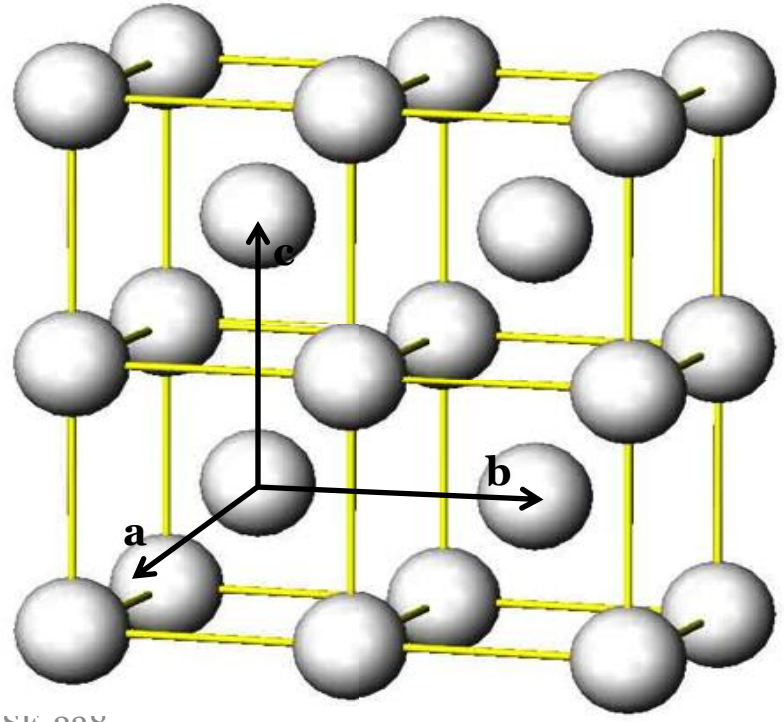
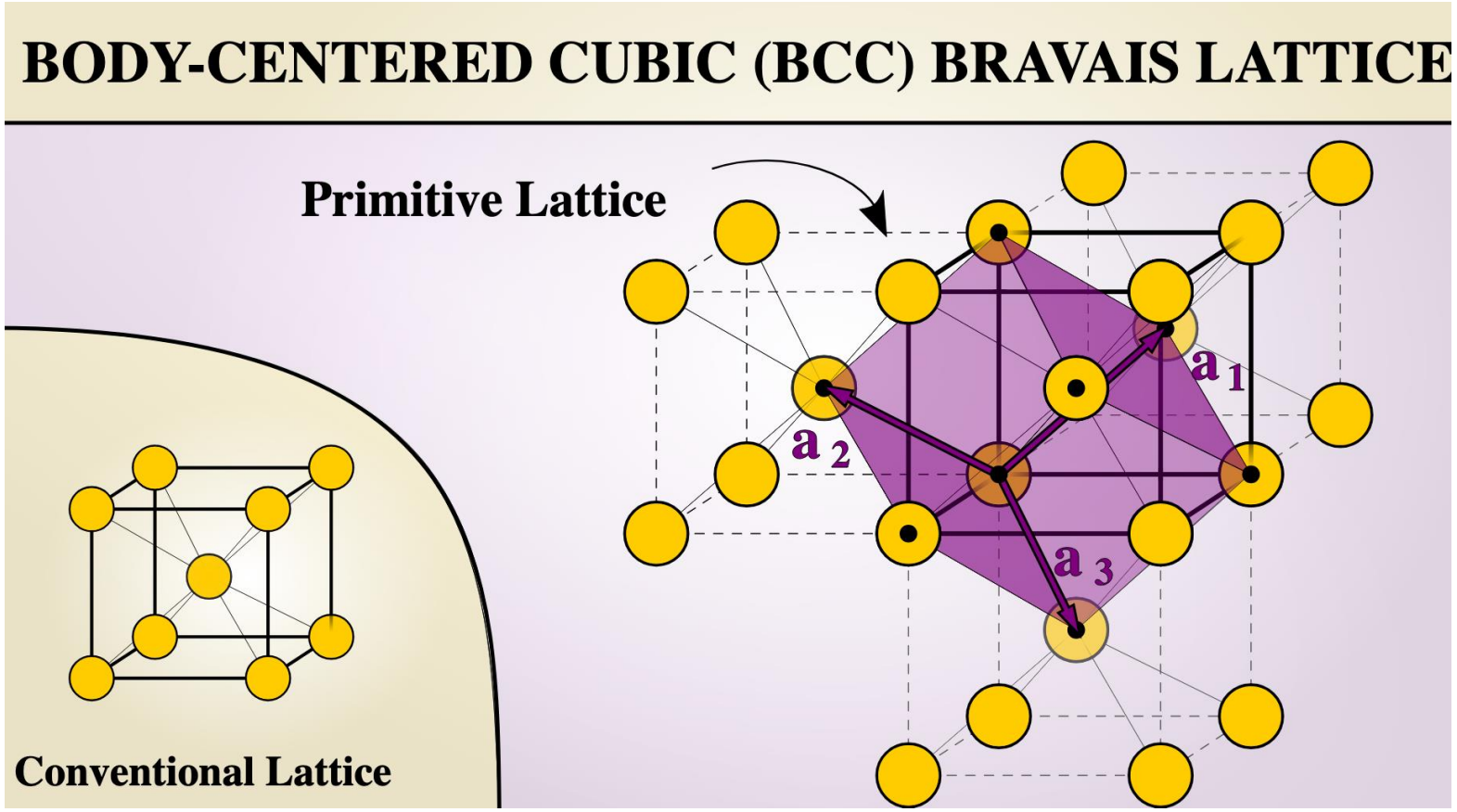


The cubic system: BCC primitive unit cell

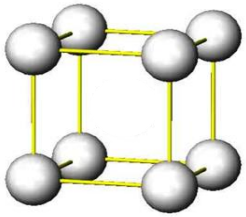


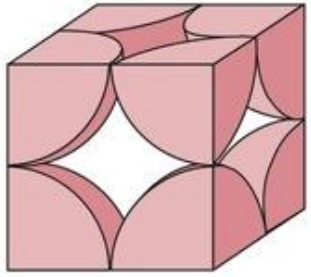
https://msestudent.com/body-centered-cubic-bcc-unit-cell/

Number of motifs per unit cell

Primitive cubic:

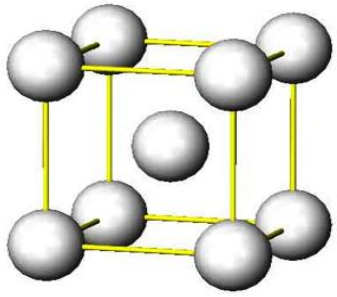
- Motifs at the corners of the cube count for 1/8
- The primitive cell has as expected 1 motif.





Body-centered conventional cell:

- Motifs at the corners of the cube count for 1/8
- The motif inside the cell count for 1.
- Hence the BCC conventional unit cell has 2 motifs.

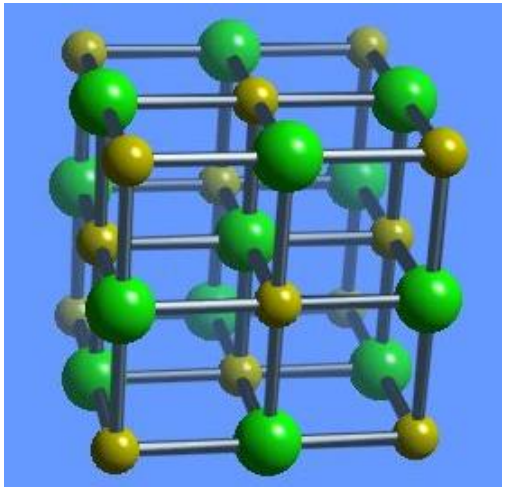


The cubic system

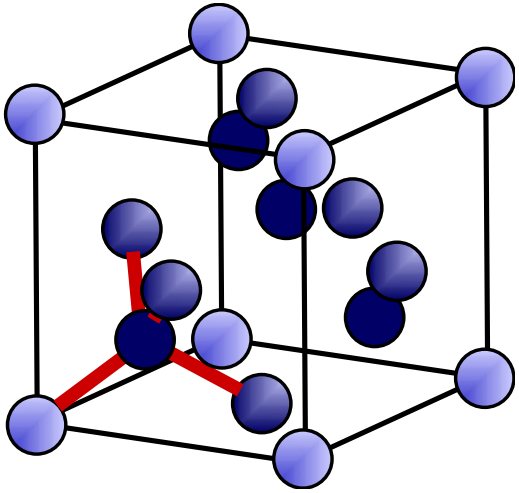
Examples:

Lattice:

Aluminium



NaCl



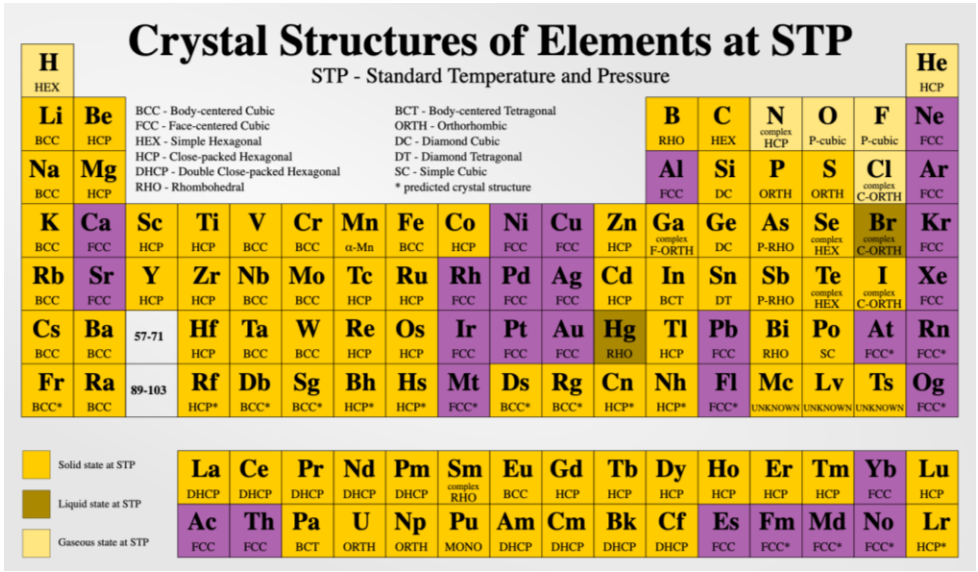
Diamond

Motifs:

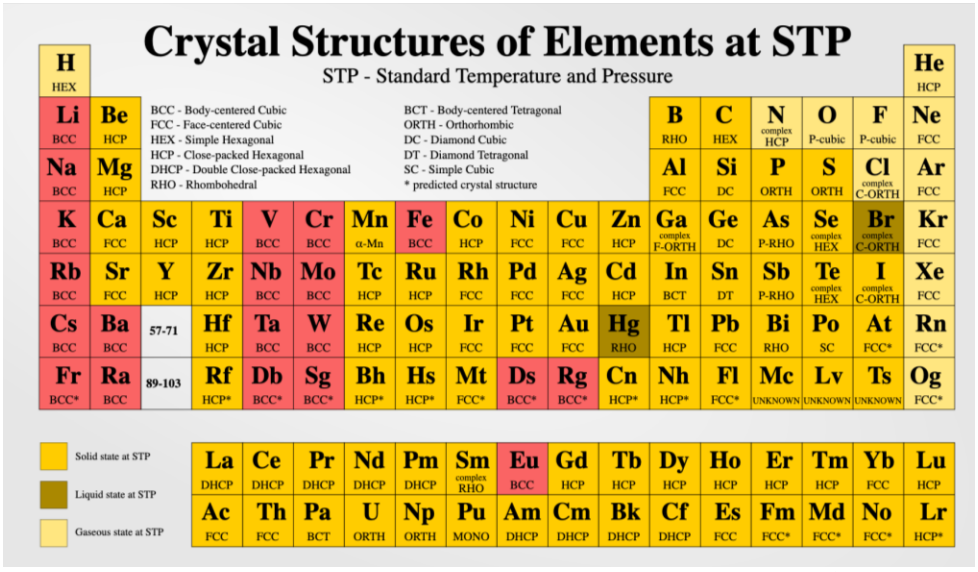
FCC and BCC

- very common crystal structure, for example for single element crystals
- a lot of metals crystalize in fcc or bcc

fcc

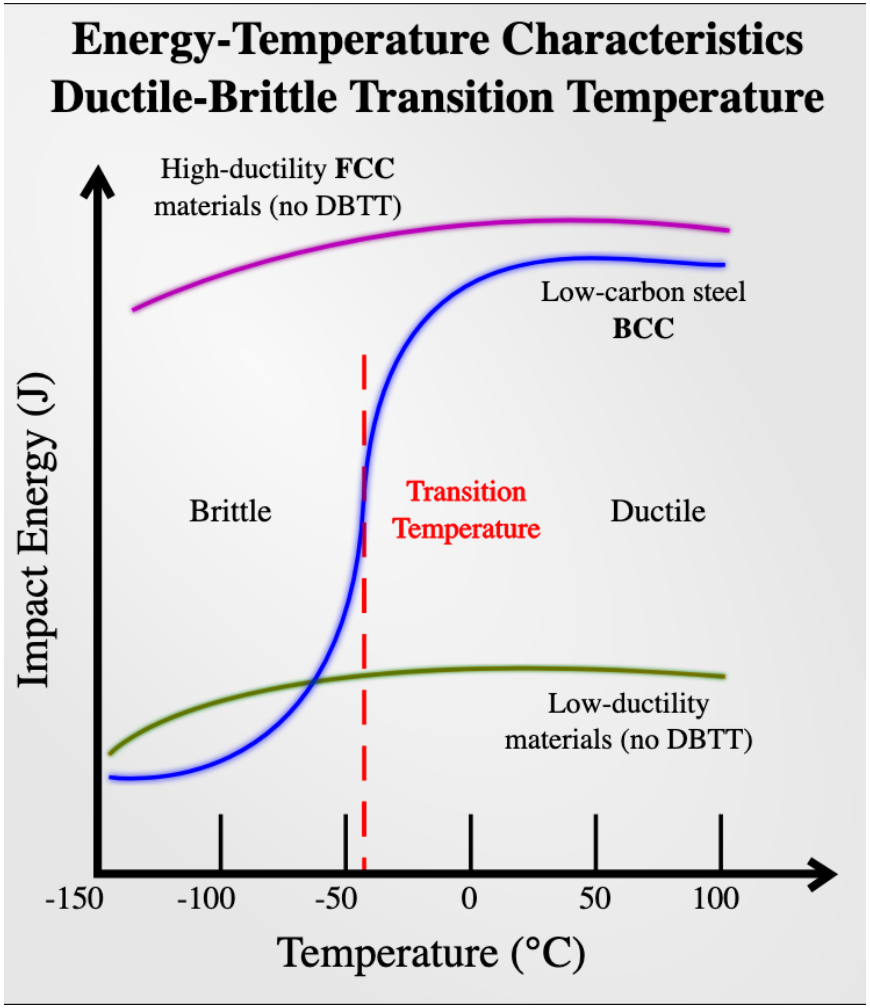


bcc



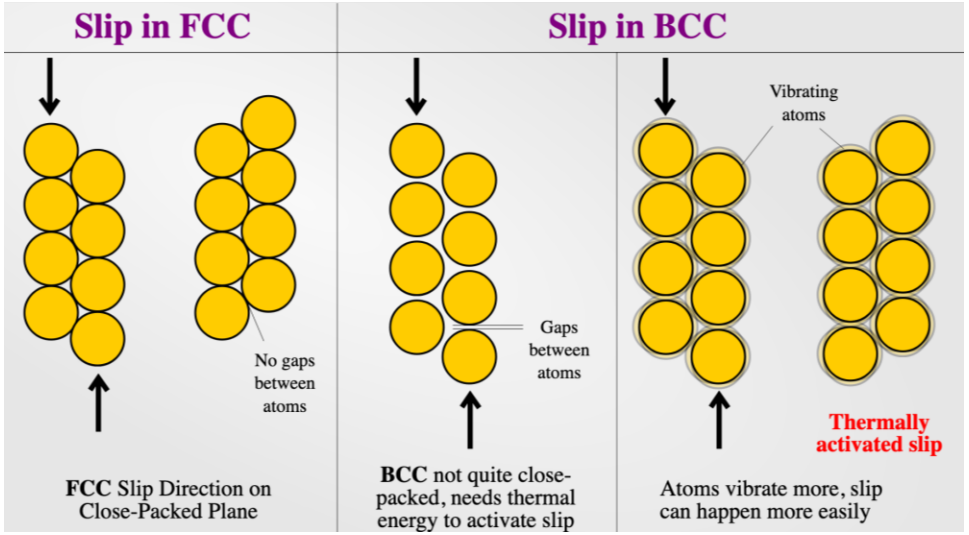
msestudent.com

Structure → Property!



FCC metals tend to be more ductile than BCC metals because of their different slip system

BCC metals have a ductile-to-brittle transition



msestudent.com

Packing density

- Represent motif as rigid spheres (hard sphere model) to give insight about the atomic arrangement and resulting properties
  - Packing factor/packing fraction $p = \frac{N_{atoms\ per\ unit\ cell} \times V_{atoms}}{V_{unit\ cell}}$

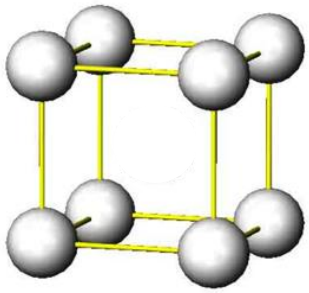
    also called compacity c

  - Density: $\rho = \frac{N_{atoms\ per\ unit\ cell} \times m_{atoms}}{V_{unit\ cell}}$

Packing density

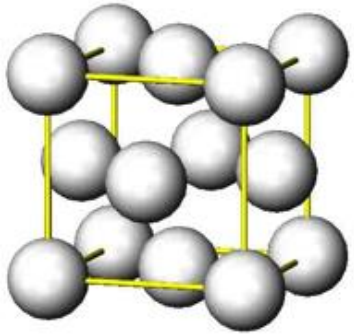
- Represent motif as rigid spheres (hard sphere model) to give insight about the atomic arrangement and resulting properties
  - Packing fraction/packing factor/compacity (last week)

primitive cubic



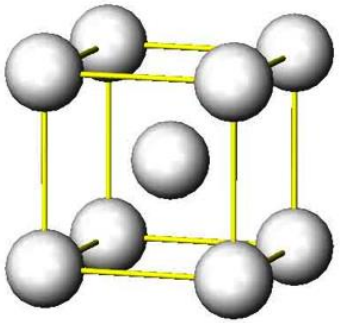
52%

face centered cubic



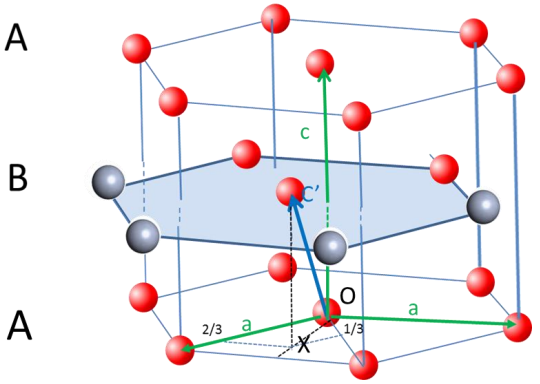
74%

body centered cubic



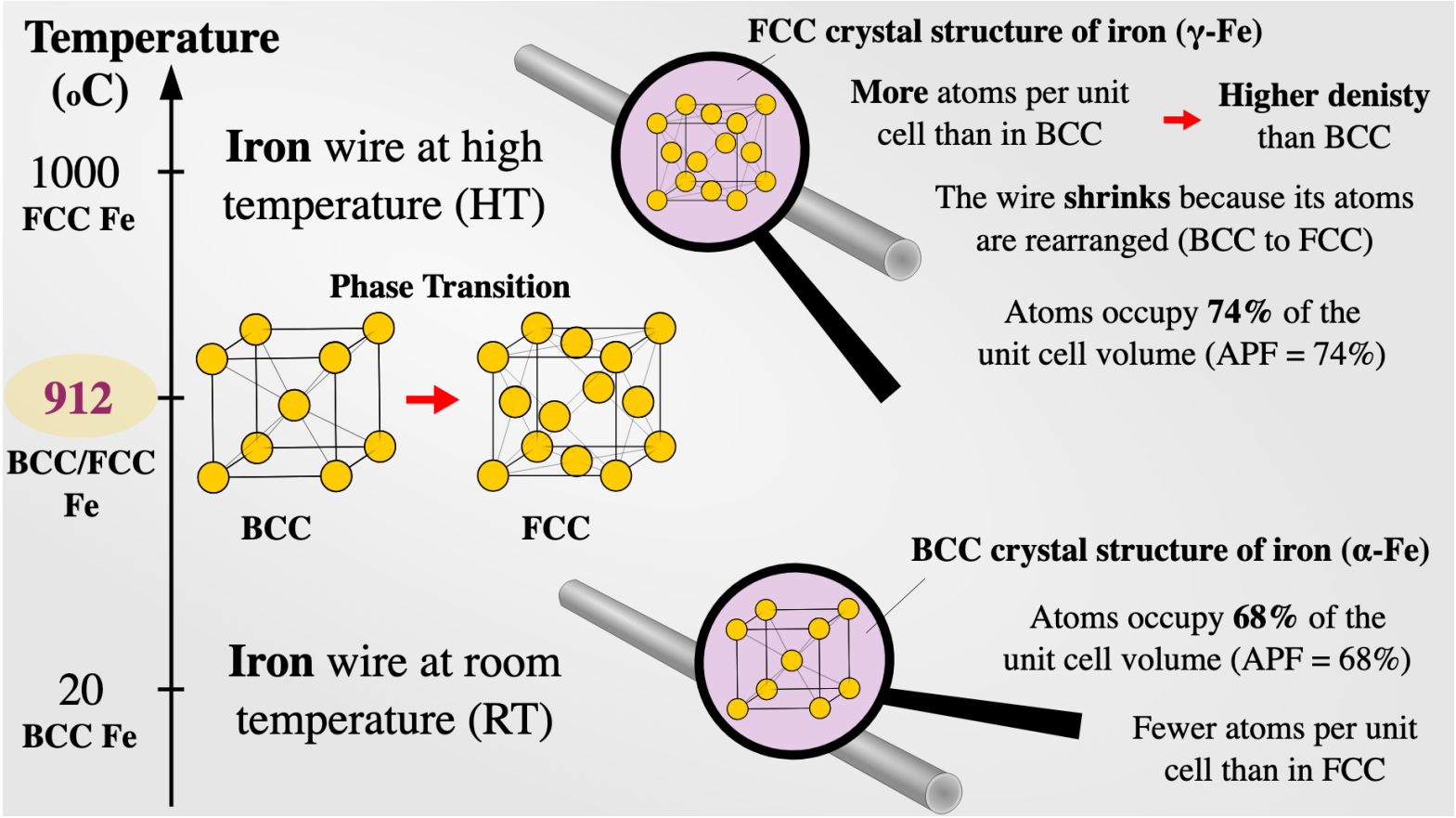
68%

hexagonal closed packed



74%

Polymorphism of Iron



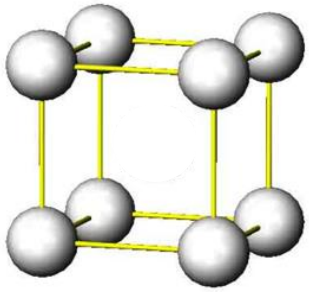
austenite

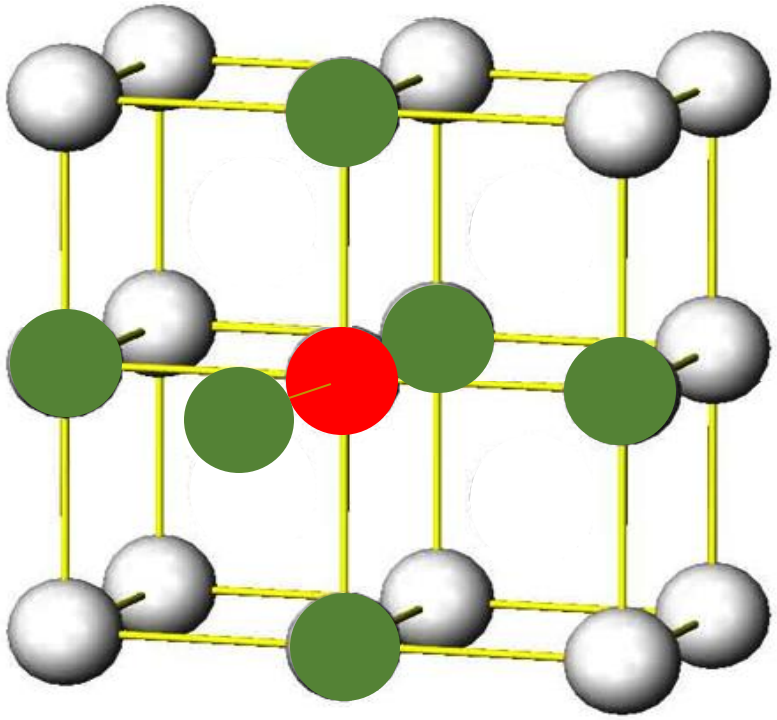
ferrite

Coordination number

- Represent motif as rigid spheres (hard sphere model) to give insight about the atomic arrangement and resulting properties
  - Packing fraction/packing factor/compacity (last week)
  - Density
  - coordination number: number of closest neighbors

for primitive cubic

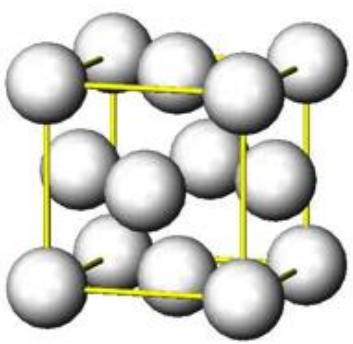


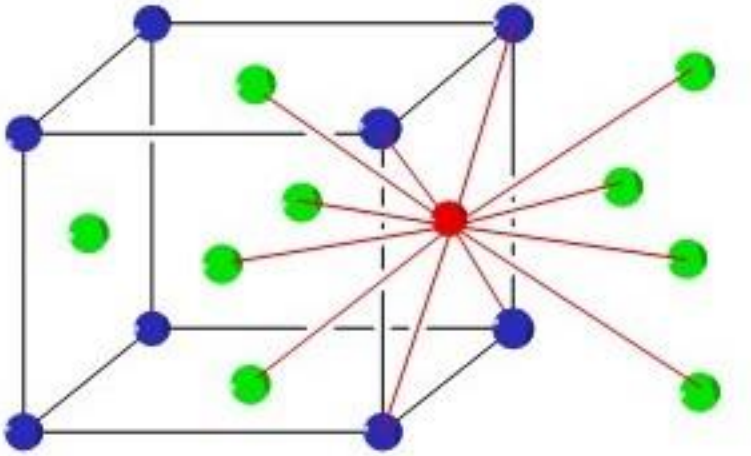


Coordination: 6

Coordination

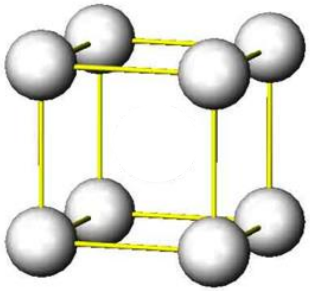
- Face-centered Cubic: Coordination: 12



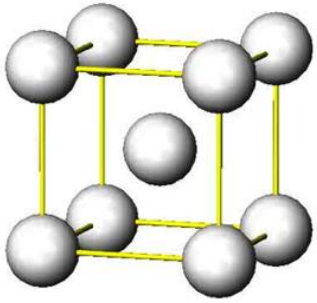


Coordination

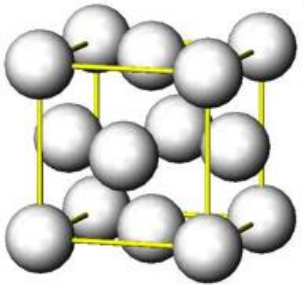
- Coordination number:



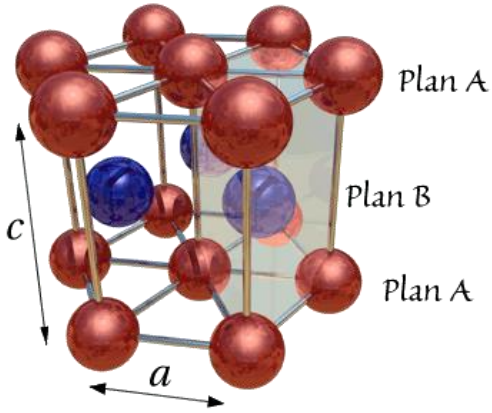
6



8



12



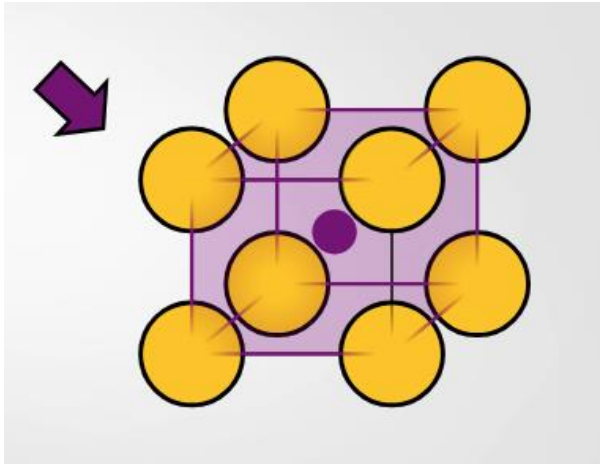
Hexagonal close-packed :

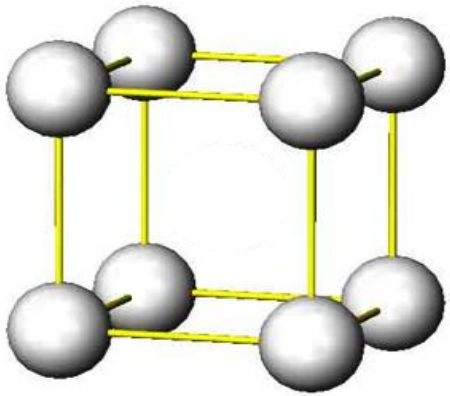
-Coordination: 12

Interstitial Sites

- Crystal structures have a certain packing density, and hence also a free volume where voids exist called interstitial sites.
- Interstitial sites play a very important role in atomic diffusion, as a material can host another smaller atom as is the case in steel (Fe+C).

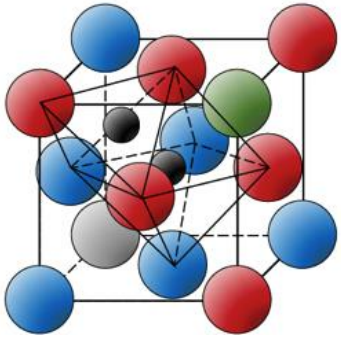
- Primitive cubic:
  - cubic interstitial site in between 8 atoms





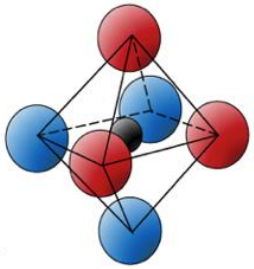
Interstitial Sites

- The FCC structure, has two main interstitial sites:



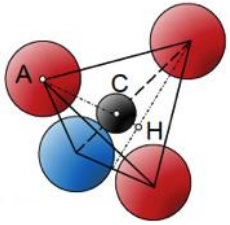
Octahedral: at the center of the conventional cell

Coordination number: 6

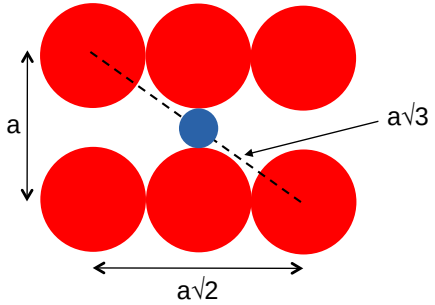


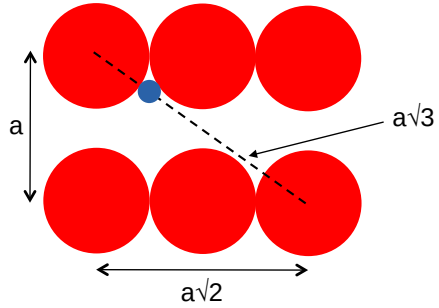
8x Tetrahedral: at 1/4 of the diagonal of the cube

Coordination number: 4



In the planes:





→ Exercise: size comparison of interstitial sites in different structures

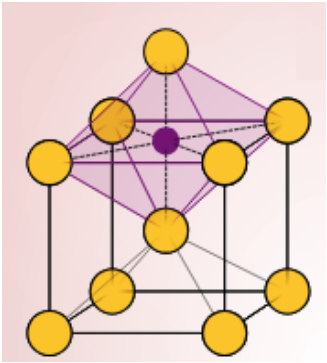
Interstitial Sites

- Interstitial sites in the BCC:

6x Octahedral: centered around each of the six BCC cell faces

e.g. center (1/2,1/2,0)

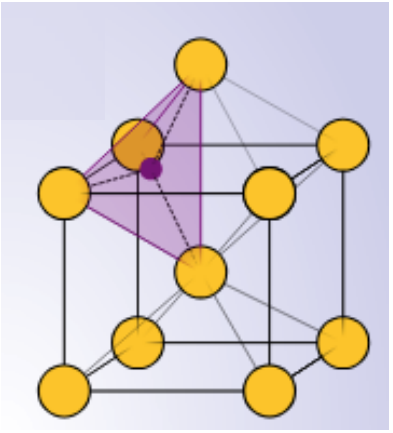
Coordination number: 6



12x Tetrahedral: at midpoint of the edges

e.g. center (1/2,1/4,0)

Coordination number: 4

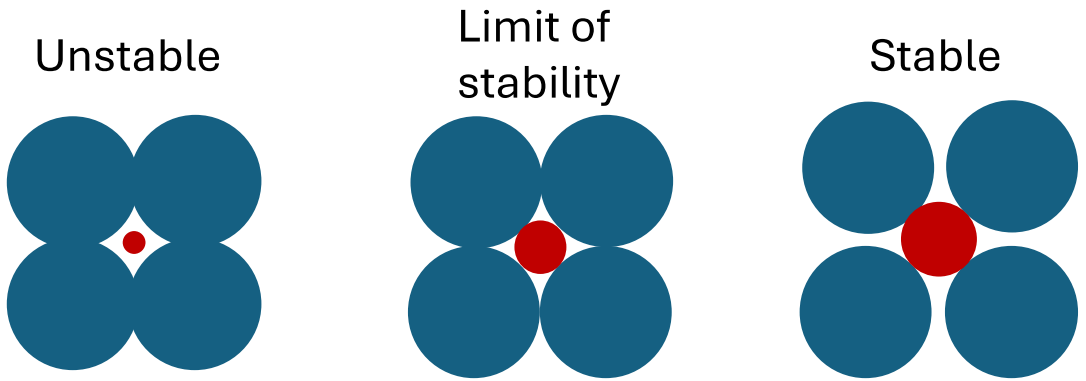


both Octahedral and Tetrahedral sites are irregular!
the coordination atom closer to the interstitial site determine what size can fit

→ Exercise on Iron FCC and BCC

Interstitial Sites: Ionic crystals

- The notion of interstitial sites can also be used to estimate what the crystal structure could be of ionic crystals.
- Different rules exist regarding the relative size of the Anions and the Cations in the molecules and their charges (take valence into account).
- For two atoms (NaCl, ClCs, ZnS etc…), a first rule called the Radius Ratio rule, or first Pauling rule, establishes a formula that defines the coordination number depending on the ratio of the radius of the cation to the one of the anion:
- $\rho = \frac{r_+}{r_-}$
- Based on the hard sphere model, it predicts that if the cation is too small, anions get too close to each other which lead to an unstable structure due to repulsive forces.



Interstitial Sites: Ionic crystals

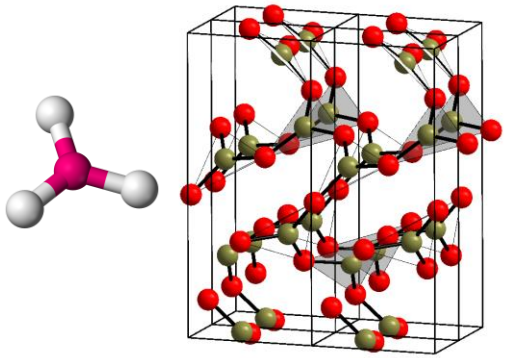
At the limit of stability, geometric consideration can give a condition on the ratio to have a certain coordination, in other words a certain number of cations surrounding an anion (and vice versa).

$$\rho = \frac{r_+}{r_-}$$

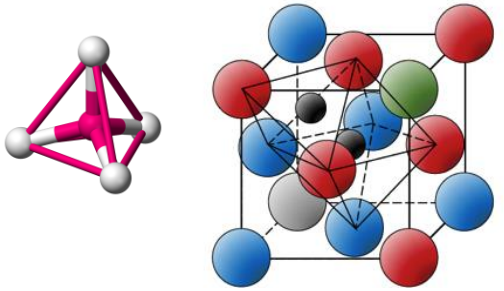
The formula is given by : $\rho = \sqrt{\frac{12}{12-CN}} - 1$, where CN = coordination number

→ Exercise for geometrical consideration

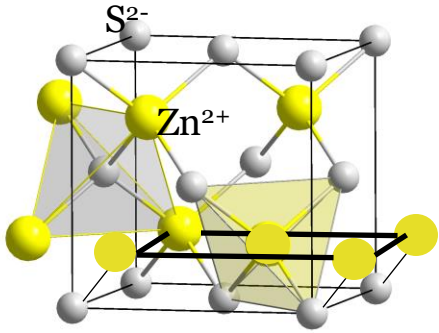
- For $0.1547 < \rho < 0.2247$,
  - $CN = 3$
  - Type of void: triangular planar
  - Example: B_2O_3 (trigonal)



- For $0.2247 < \rho < 0.4142$,
  - $CN = 4$
  - Type of void: Tetrahedral
  - Example: ZnS, CuCl, GaAs



Void in FCC



Zincblende structure

Two interpenetrating FCC structures shifted along ¼ of the diagonal

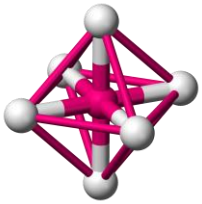
Interstitial Sites: Ionic crystals

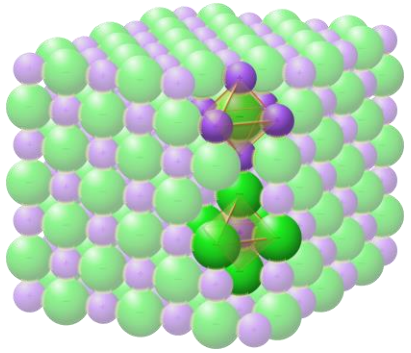
For $0.4142 < \rho < 0.7320$:

$CN = 6$

Type of void: Octahedral

Example: NaCl, MgO



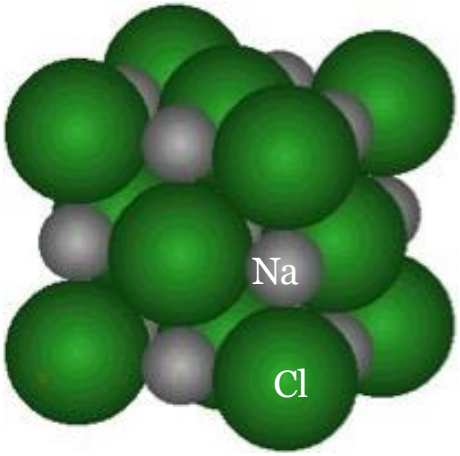


Rock salt structure

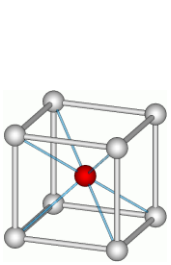
Na Cl

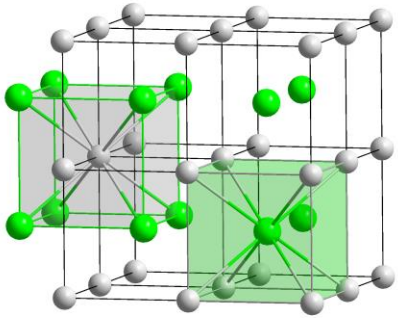
R_{Na+} = 1.02 Å

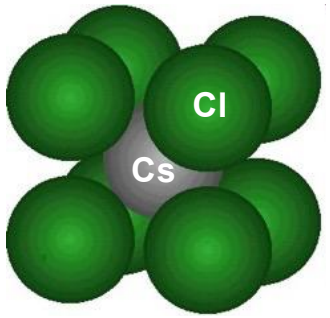
R_{Cl-} = 1.81 Å



- For $0.7320 < \rho < 1$:
  - $CN = 8$
  - Type of void: Cubic
  - Example: CsCl







CsCl structure

Cs Cl

R_{Cs+} = 1.67 Å

R_{Cl-} = 1.81 Å

Ionic crystals

- The ratio rule presented above enables to predict the crystal structure fairly well given its simplicity: in some studies, in a set of over 5000 oxides, 66% of coordination environments agree with Pauling's first rule.
- Many exceptions do exist: Oxides formed with alkali or alkali-earth meta, or cations that contain multiple cation coordination are common deviations from this rule.
- The discrepancy lies also mostly in the polar, and not strictly ionic, nature of bonds inside the crystals.
- There are other rules, such as, for example:
    - The electrostatic valence rule looks at the strength of the electrostatic force as a function of the charge and the coordination number.

Cations with oxide O^{2-} ion

Cation	Radius ratio	Cation C.N.	Electrostatic bond strength	Anion C.N.
Li^{+}	0.34	4	0.25	8
Mg^{2+}	0.47	6	0.33	6
Sc^{3+}	0.60	6	0.5	4

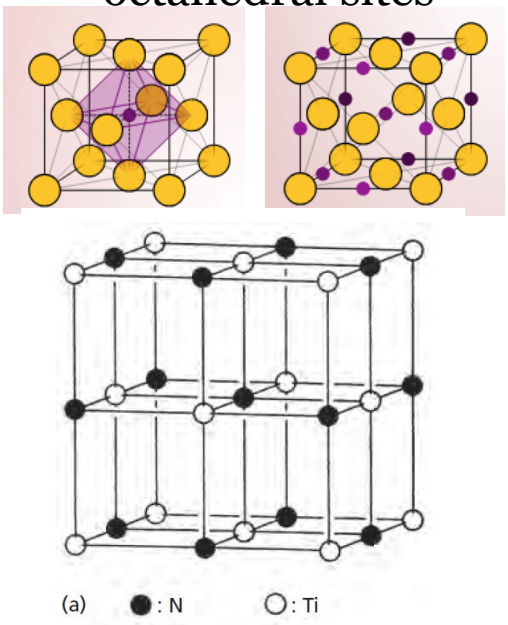
- Other geometric rules exist but they are accurate only in 13% of cases, limiting their use.

Interstitial compounds

- Metal hydrides, nitrides, borides, carbides, etc., in which the radius ratio of the (small) non-metallic atoms to the (large) metal atoms is small, provide good examples of interstitial compounds.
- However usually the interstitial atoms are oversized (in terms of radius ratio) and push apart the structure → not strictly closed packed

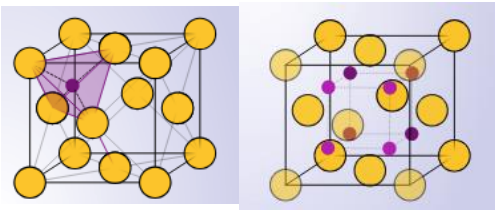
Example: FCC

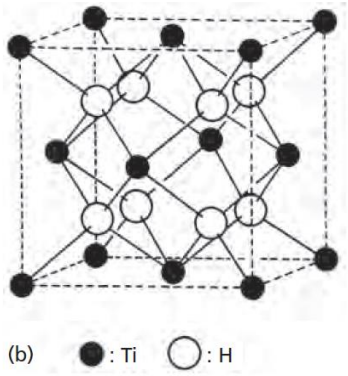
octahedral sites



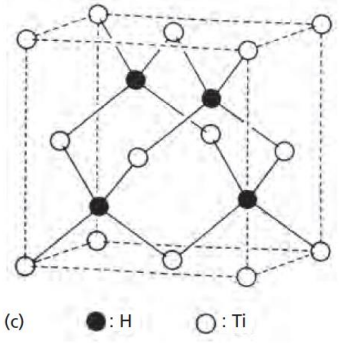
TiN: hard ceramic coating

tetrahedral sites





TiH_2

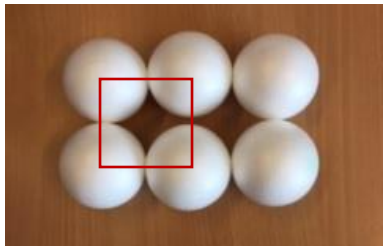


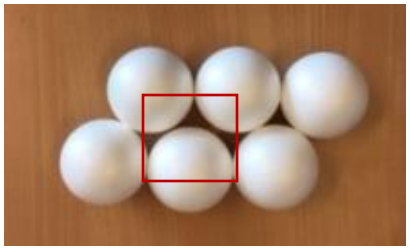
TiH

brittle and hard, powder

The hard sphere model

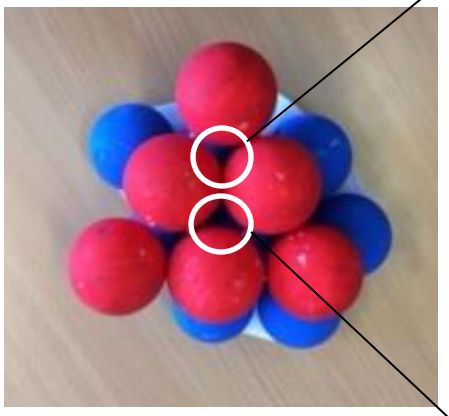
- 2D configuration:

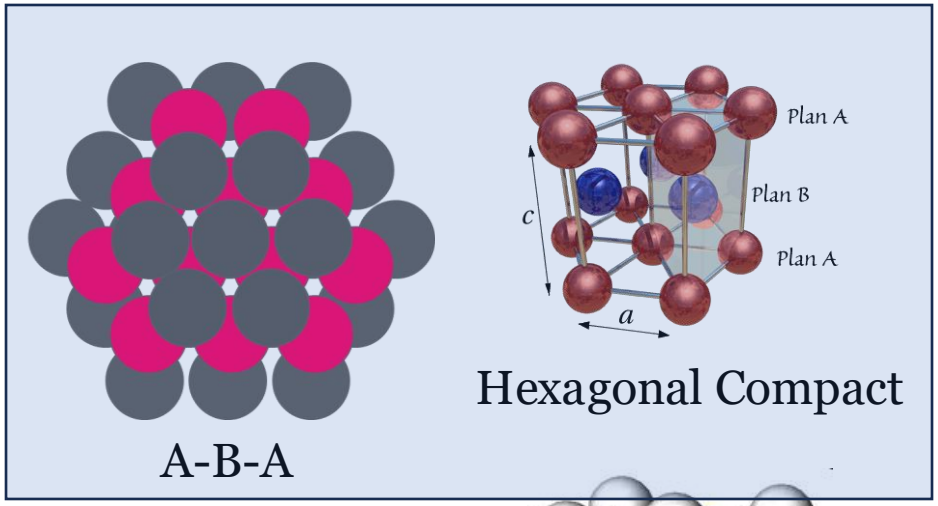


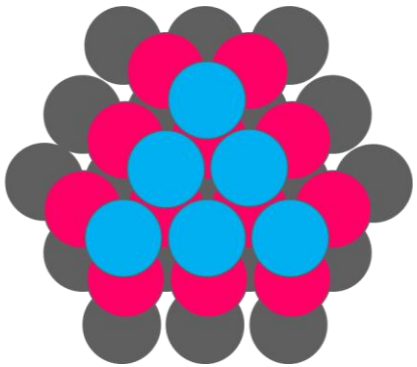


- 3D configuration:

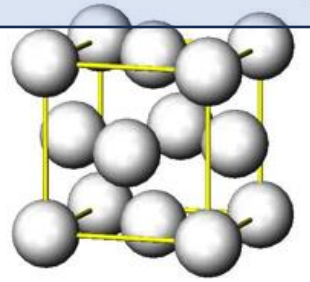
2 possibilities:







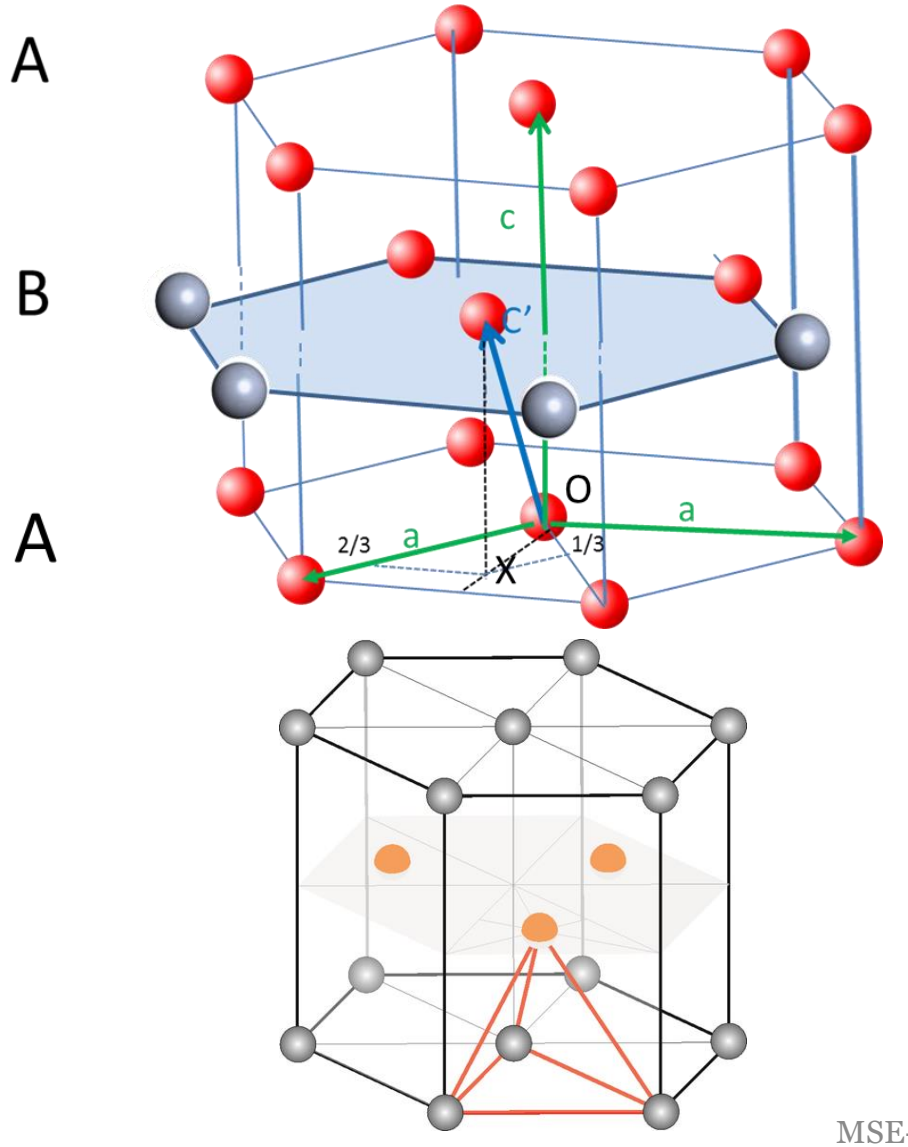
A-B-C



Face-centered Cubic

(= cubic-closed packed)

Hexagonal compact



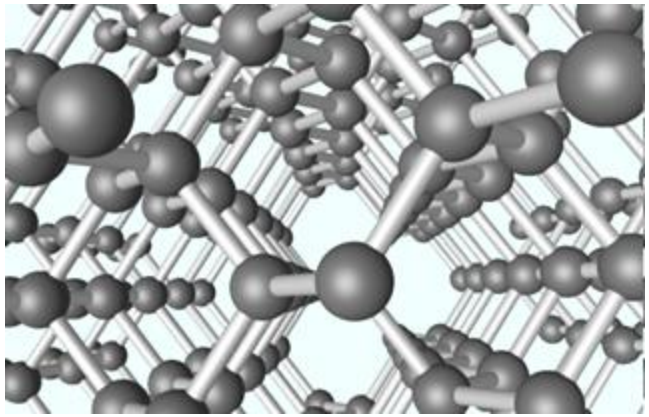
Unit cell

- Hexagonal unit block with axis a,b,c (a=b) and 2-atoms assigned to each of the corners of the unit block: coordinates (0,0,0) and (2a/3 , a/3, c/2)
- Primitive unit cell a,b,c'
- Hexagon with three atoms in the center.

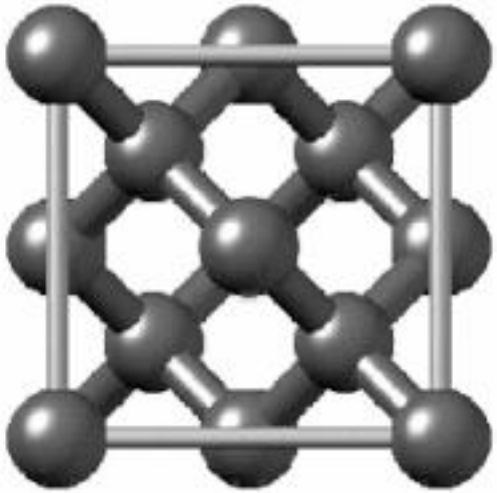
Covalent crystals and covalent/ionic crystals

- While ionic crystals and crystals with metallic bonds usually take closed packed forms, the structure of covalent crystals is dictated by the directional covalent bond and the configuration of the atomic orbitals
- high strength of covalent and ionic bonds: high hardness

"pure" covalent crystals:
Diamond, Silicon
sp^3 orbitals → tetrahedrally
coordination number: 4



from Exercise week 1:
packing fraction only 34%



http://en.wikipedia.org/wiki/Diamond_cubic

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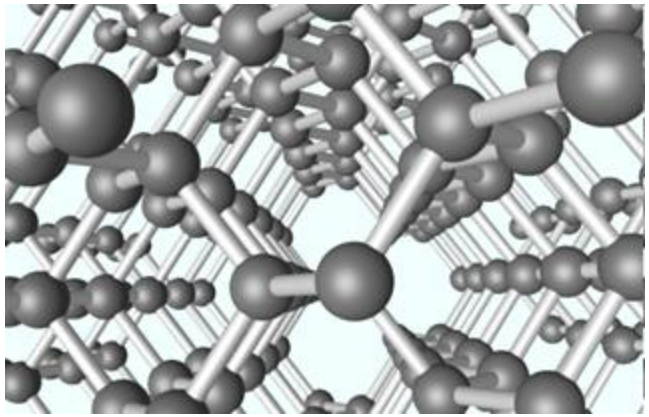
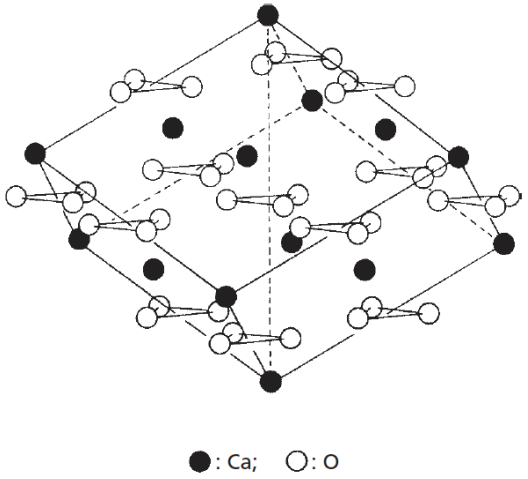
sp^3 orbitals → tetrahedrally
coordination number: 4



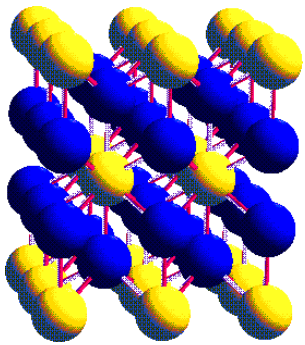
from Exercise week 1:
packing fraction only 34%

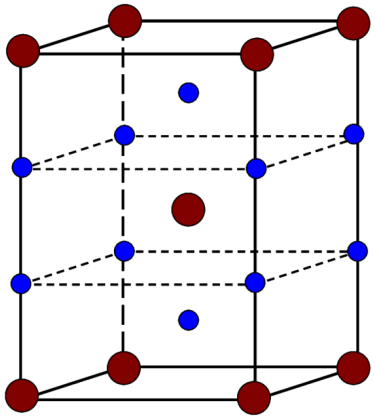
many inorganic crystals
mixture of covalent and ionic bonds
$Ca^+ (CO_3)^-$



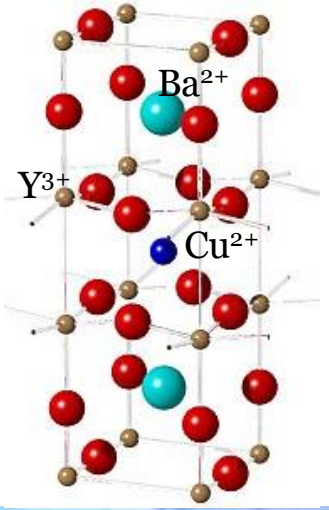
Ceramics

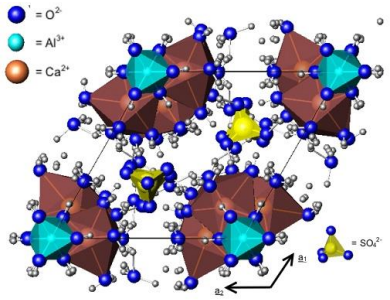
- Ceramics possess ionic or covalent (or polar) bonds that are very strong.
- The structure can be compact like metals but more complex, as it depends on the ionic radius of the different atoms, and their valence.
- As a result, the crystallographic arrangements can be quite complex and they have a **higher ability to be quenched into an amorphous structure.**





$MoSi_2$

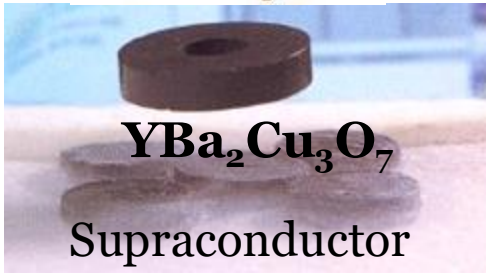




Ettringite structure

structure formed in the early phases of hardening of **cement**

Resistive heating elements



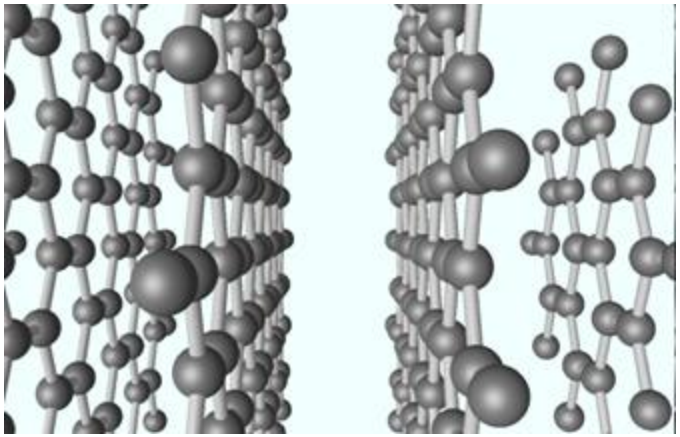
$YBa_2Cu_3O_7$

Supraconductor

Covalent and VDW crystals

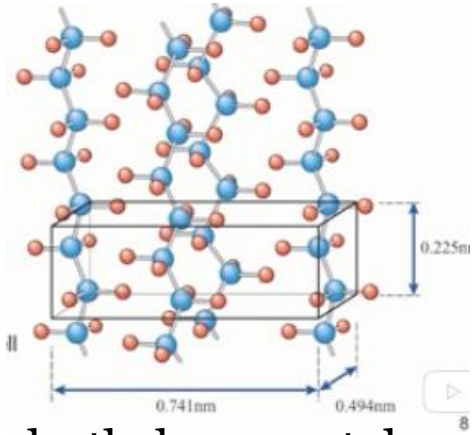
- Layered inorganic structures: covalently bond within a layer, van der Waal bonds between the layers: graphite, talc, mica
- Organic crystal: covalent bonds within molecule, van der Waal (and H-bonds) in between

covalent bond
sp_2 orbital → planar
in between Van der Waal bonds



graphite
→ anisotropic mechanical properties

organic crystals
molecules strongly bond
if they can be arranged regularly form crystal

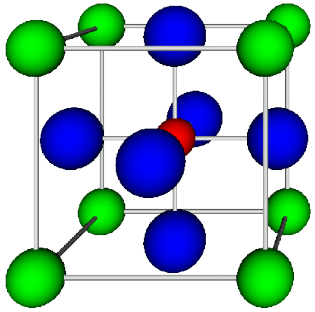


Polyethylene crystal
→ soft materials
→ more in polymer lecture

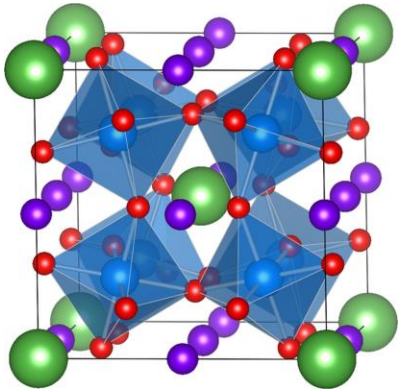
Examples for more complex crystals

Perovskites

ABX3, where:
- **A** is a large cation
- **B** is a smaller metal cation
- **X** is an anion

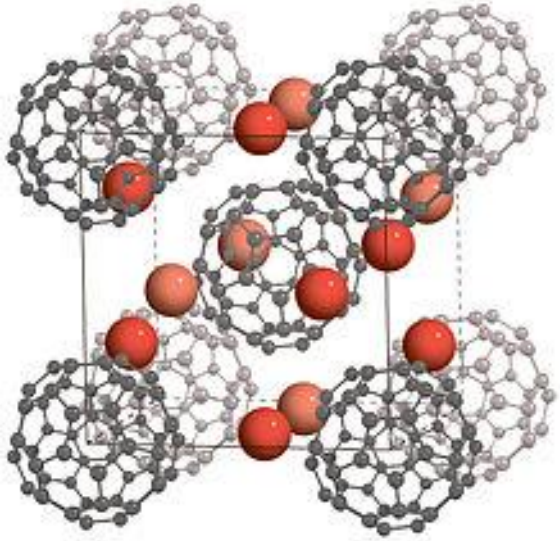


$CaTiO_3$ (Ca=green, O=blue, Ti=red)



Double perovskite structure $AA'_3B_4O_{12}$.

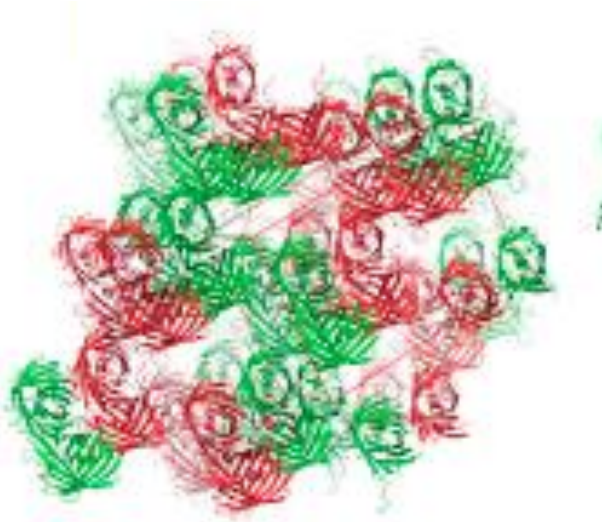
Molecular crystals



alkaline-metal-doped fullerenes such as Cs_3C_{60} crystals superconductive at 33K

http://en.wikipedia.org/wiki/Buckminsterfullerene

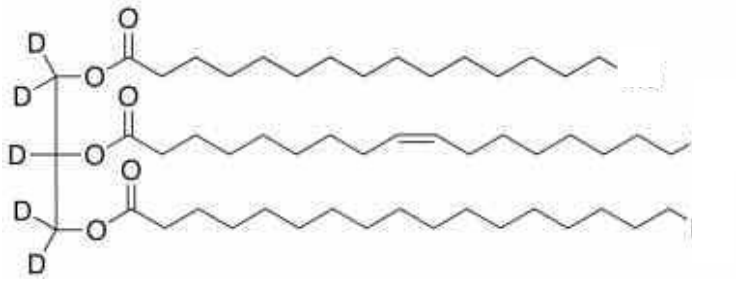
Protein crystal



photoactivatable fluorescent protein h41pkve6

Organic crystal: Chocolate

polymorphism of Triacylglycerol (= cocoa butter)



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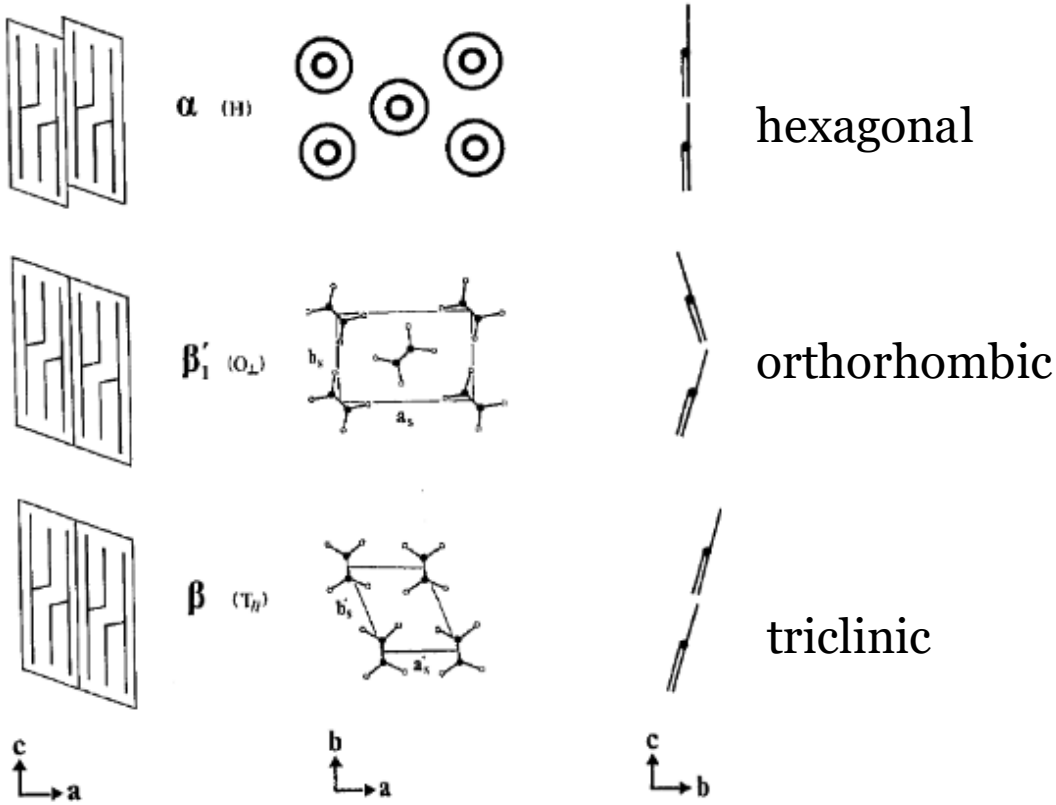
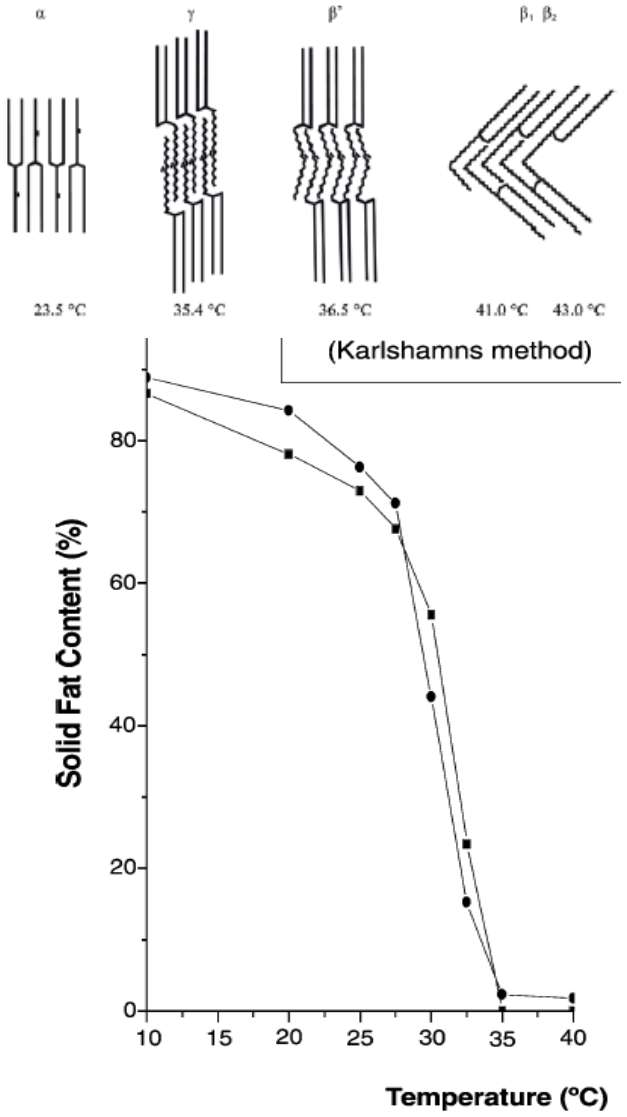


Chocolate

- cacao butter, main component POS and many other triglycerides
- polymorphism: 6 different crystalline forms: γ, α, III, β_{IV}, β_V, β_{VI}
- β_V: desired form due to melting behaviour, gloss, texture and breaking behaviour
  BUT: is not the thermodynamically most stable form
- fat bloom related to recrystallization
  $\beta_V \rightarrow \beta_{VI}$ transition



Crystal symmetry –> outer shape?

Patterns in Art and Science

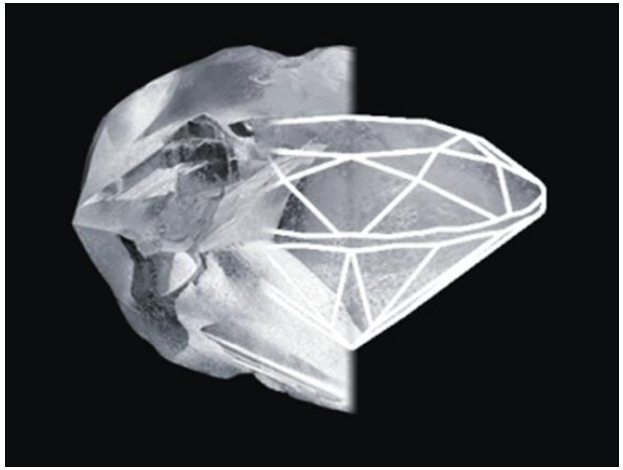
exhibition at the EPFL Pavilions until March 9!

hexagonal aquamarine beryl crystal

Outer shape and symmetry

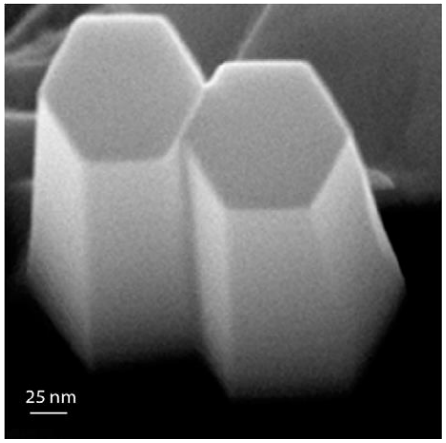
Naturally grown quartz crystal

http://en.wikipedia.org/wiki/Quartz



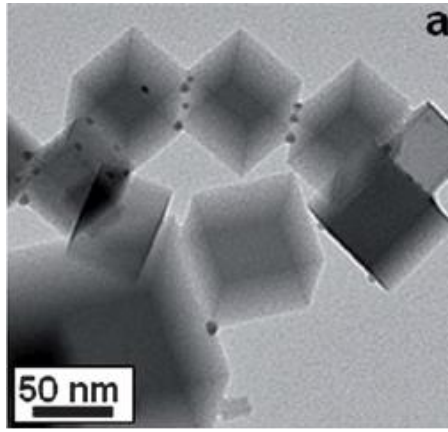
Particular crystal shape cut out from larger naturally grown crystal

http://www.ogitechusa.com/images/sawcut/3.png



SEM image of GaAs nanostructure revealing 6-fold symmetry

Mohan et al, Nanotechnology 23 (2012) 025601

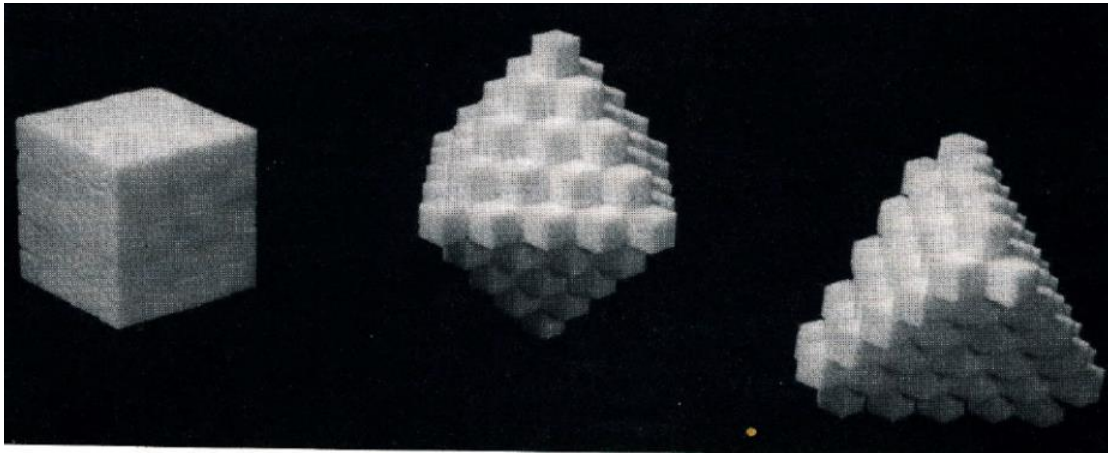


TEM image of MgO crystals

Stancic et al, Nanoscale, 2013, 5, 2448

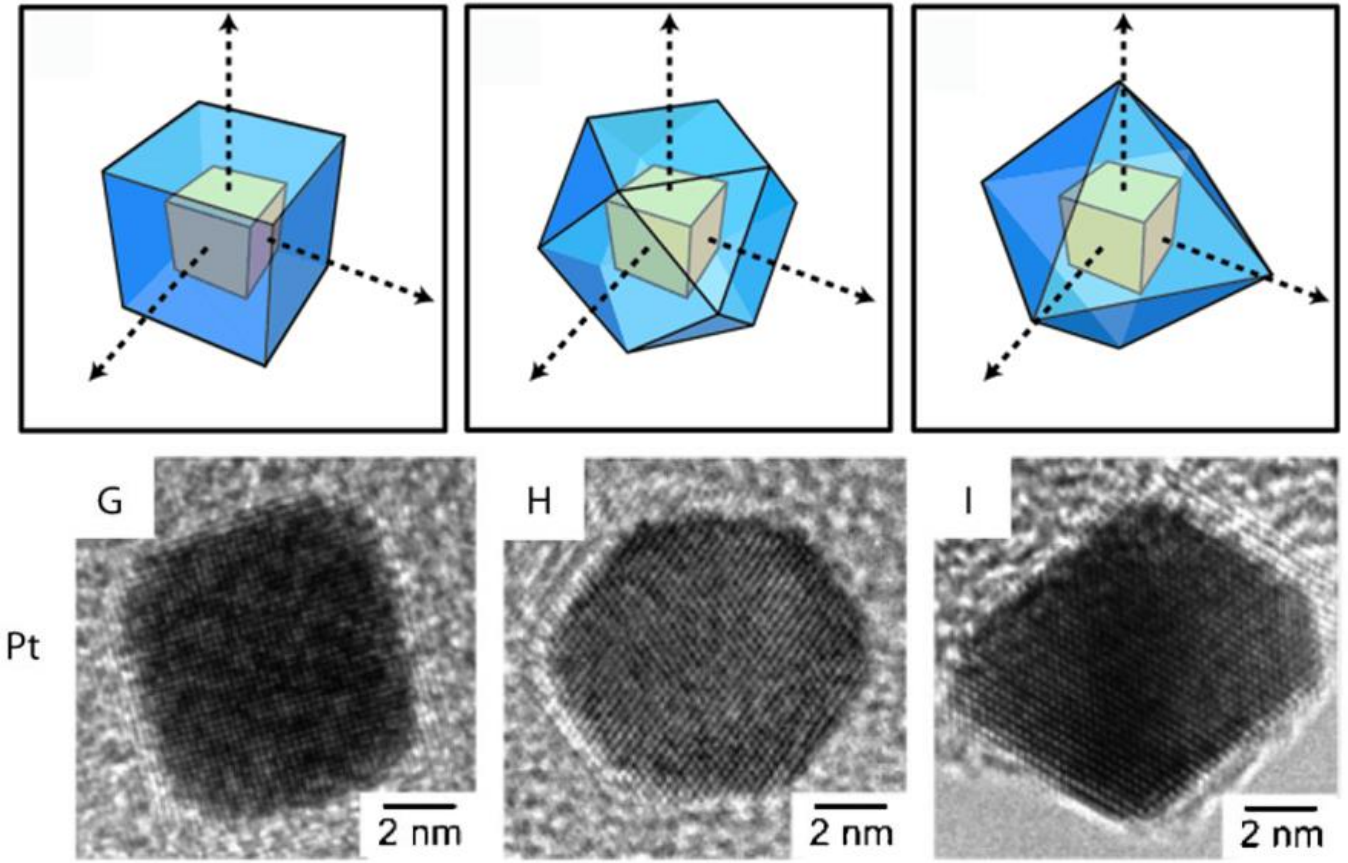
The shape of crystals is related to the nature of the internal order and symmetry in which atoms are arranged. For instance, the quartz crystal has grown in the form of six-sided prisms with a six-sided pyramid at the end. Shape of a crystal can be already present at much smaller length scales.

Outer shape and symmetry



One can however not always rely on the outer shape of the crystal to derive symmetry.
The morphology of a crystal strongly depends on how and how many building blocks are put together, as illustrated schematically below in an image taken from Hammond (fig.4.1).

Outer shape and symmetry



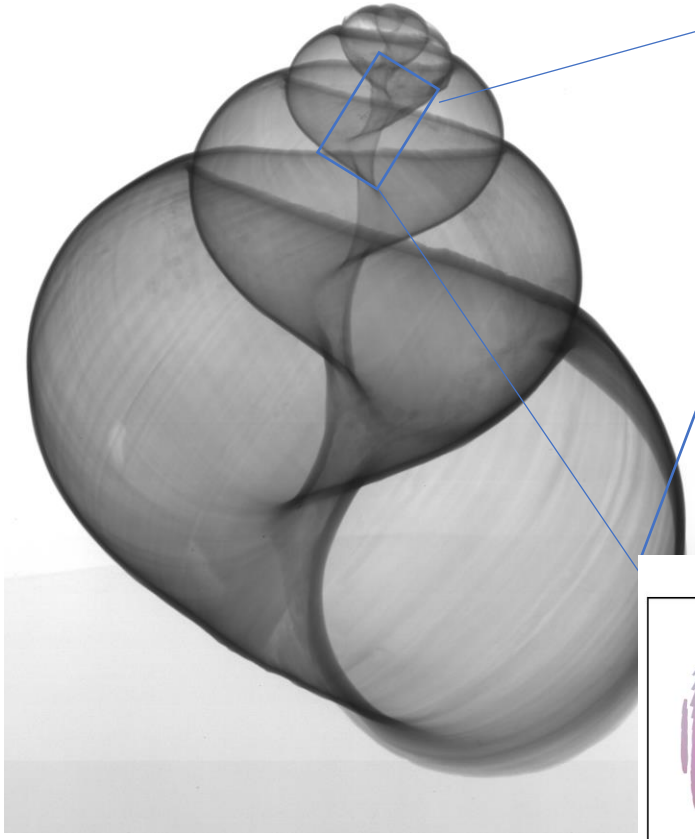
Pt nanocrystals with different shapes

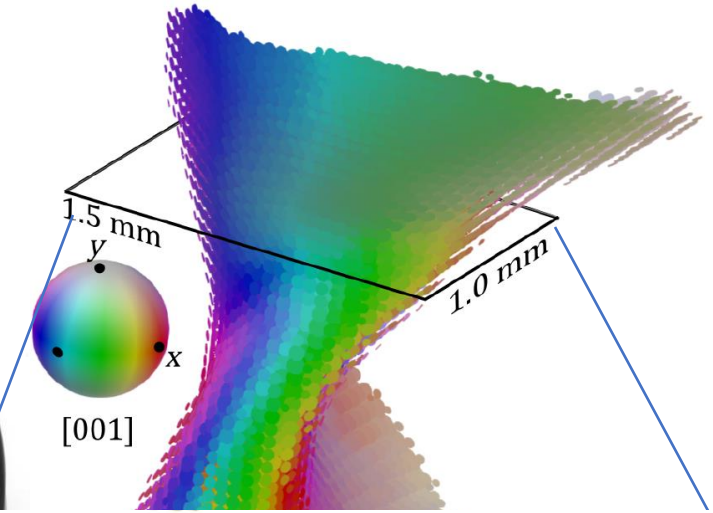
Which outer shape is reached in nanoclusters, depends for instance on the environment in which the clusters are grown and on the incorporated impurities. Controlling the morphology and the three-dimensional arrangement of atoms in crystalline nanoparticles is important for applications: for instance the morphology influences the catalytic properties of the cluster.

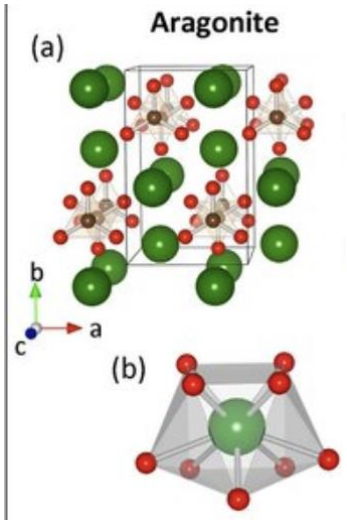
Tao et al (review paper), Small 2008, 4, No. 3, 310 – 325

Outer shape and symmetry:Biomineral

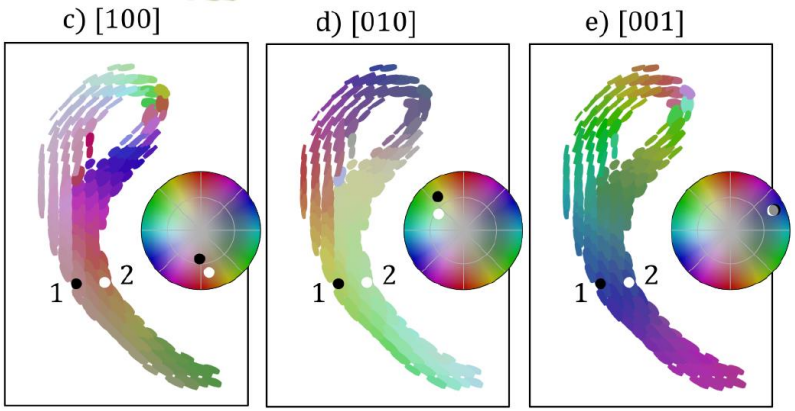
Roman snail shell







aragonite orthorombic

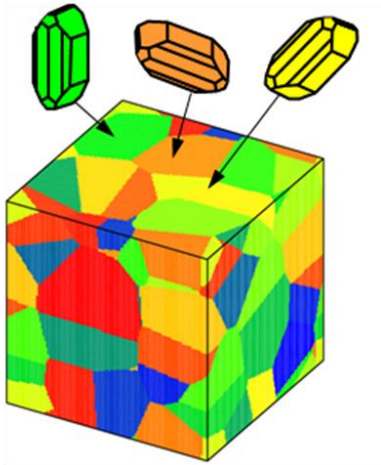


M. Carlsen *et al*. Applied Crystallography 2025

Polycrystallinity

- Single crystals form only under special conditions and it is not so easy to grow them artificially.

- Most engineering materials are not single crystals, but polycrystals. As the name suggests, a polycrystalline material is an ensemble of many crystals. The individual crystallites are often referred to as grains and the junctions between these grains are called grain boundaries. This is schematically presented below. Parameters such as the size distribution of the grains, the morphology of the grains, the orientation relationships between the individual crystals are very important parameters determining the overall properties of the polycrystalline material.



Polycrystal where each color represents a grain of the same crystal structure but with a different orientation.
(http://pimm.paris.ensam.fr/en/node/1457)

Summary

- Definition of crystal, lattice and motif
- first introduction to symmetry, crystal systems and Bravais Lattice
  →main topic for next week!
- The cubic system with its three Bravais lattice (primitive cubic, bcc, fcc)
- primitive vs. conventional cells
- Coordination number
- Interstitial Sites
- Ionic crystal and the radius ratio rule
- Covalent crystals
- Symmetry and outer shape